

A Conflict-Aware Learning Approach to SCA Verification for MAC Architectures

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Motivation

Formal Verification & Correctness

Ensures 100% functional correctness, crucial for avoiding expensive real-world defects like the **Pentium BUG**.

The Challenge

Complex arithmetic (Multiplication, MAC) is challenging for traditional formal methods:

- **SAT Solvers:** Struggle with propositional logic of multipliers.
- **BDDs:** Suffer from exponential size explosion.

SCA Limitations

Symbolic Computer Algebra (SCA) is suited for MAC verification but faces a critical hurdle:

Intermediate Polynomial Explosion

Intermediate terms grow excessively, exceeding memory limits for complex architectures.

Goal: Mitigate Term Explosion through Conflict-Aware Learning

Overview

Goal

Formal Verification of Integer MAC circuits (AxB+C)

Circuits & Experiments

- Known architectures & synthesized designs
- Improvement over state-of-the-art

Verification Technique: SCA

Optimizations:

- Dynamic Ordering
- Polarity Optimization
- **Conflict Optimization**

SCA Terminology:

$$3 ab + 4 cf - 3 ef$$

Polynomial

Monomial (e.g., 3ab)

Coefficient: (e.g., 3)

Variable: (e.g., a, b)

Agenda

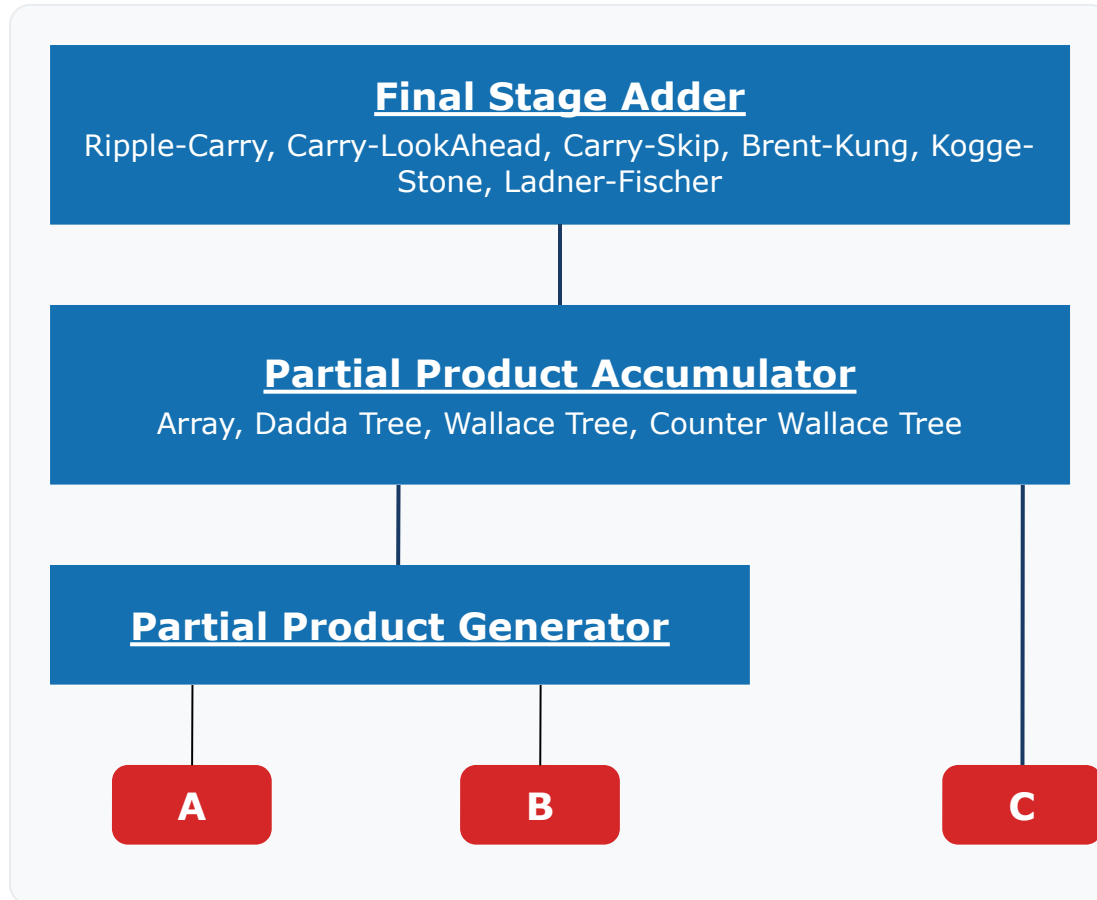
01 MAC Architectures

02 SCA Verification

03 Conflict Optimization

04 Experiments on various MAC architectures

MAC Architectures



Merged Arithmetic

- MAC is more than just MUL + ADD
- MAC: Inputs of C are embedded directly into partial products

Block Complexity

- **Partial Product Generator:** Requires n^2 AND gates
- **Partial Product Accumulation:** Compression achieved using Full and Half Adders
- **Final Stage Adder:** Performance is strongly architecture dependent

Symbolic Computer Algebra(SCA)

Example:

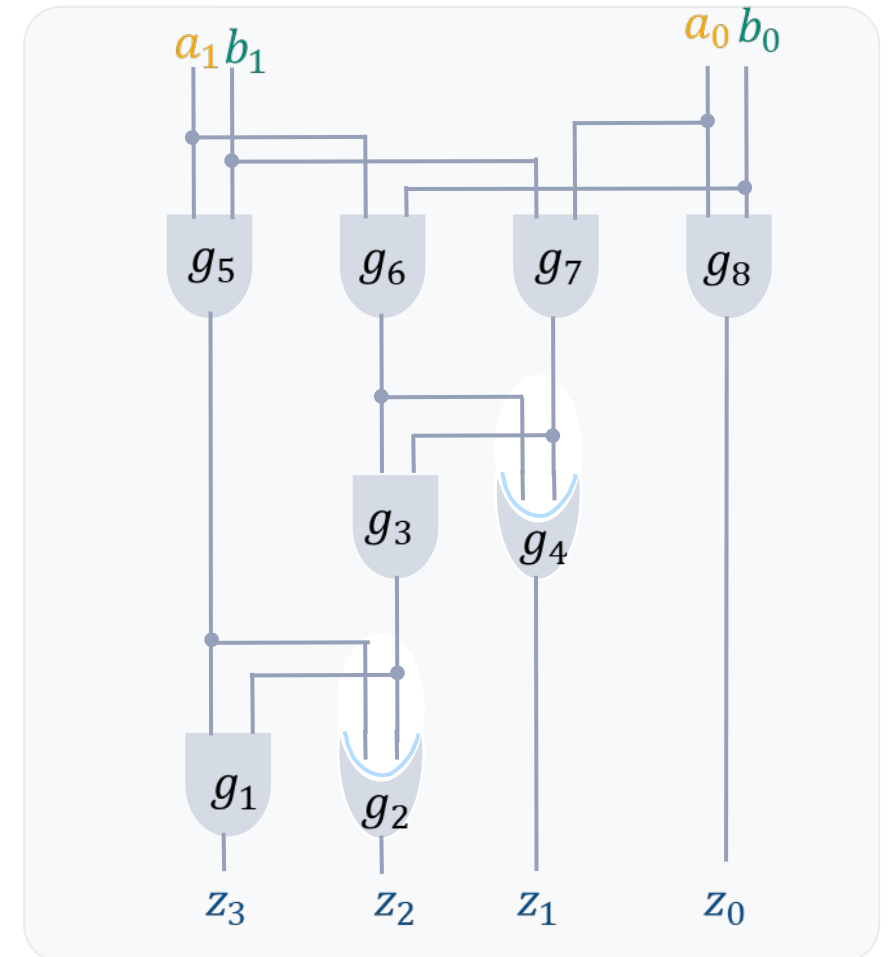
- Verification of a simple 2-bit multiplier
- Factors are represented as
 - $A = (2a_1 + a_0)$
 - $B = (2b_1 + b_0)$
- Result is represented as $Z = 8z_3 + 4z_2 + 2z_1 + z_0$

Observation:

$$Z = A \times B \rightarrow Z - A \times B = 0$$

$$8z_3 + 4z_2 + 2z_1 + z_0 - (2a_1 + a_0) \cdot (2b_1 + b_0) = 0$$

Specification
Polynomial



Symbolic Computer Algebra(SCA)

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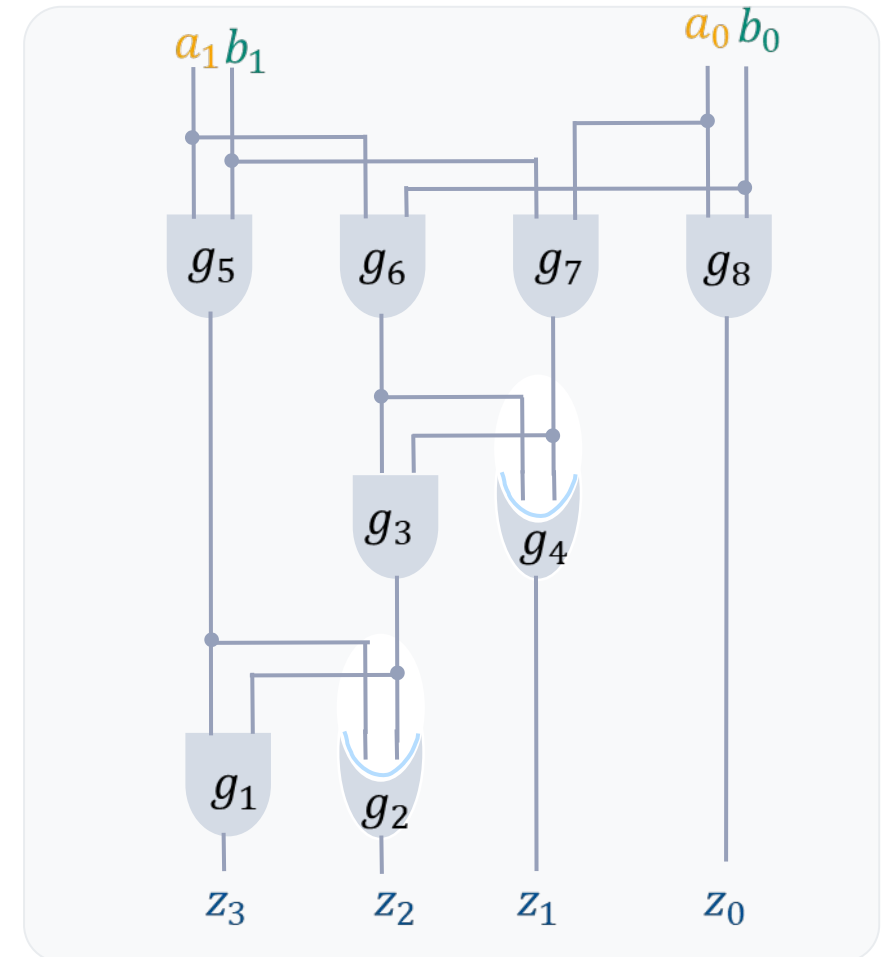
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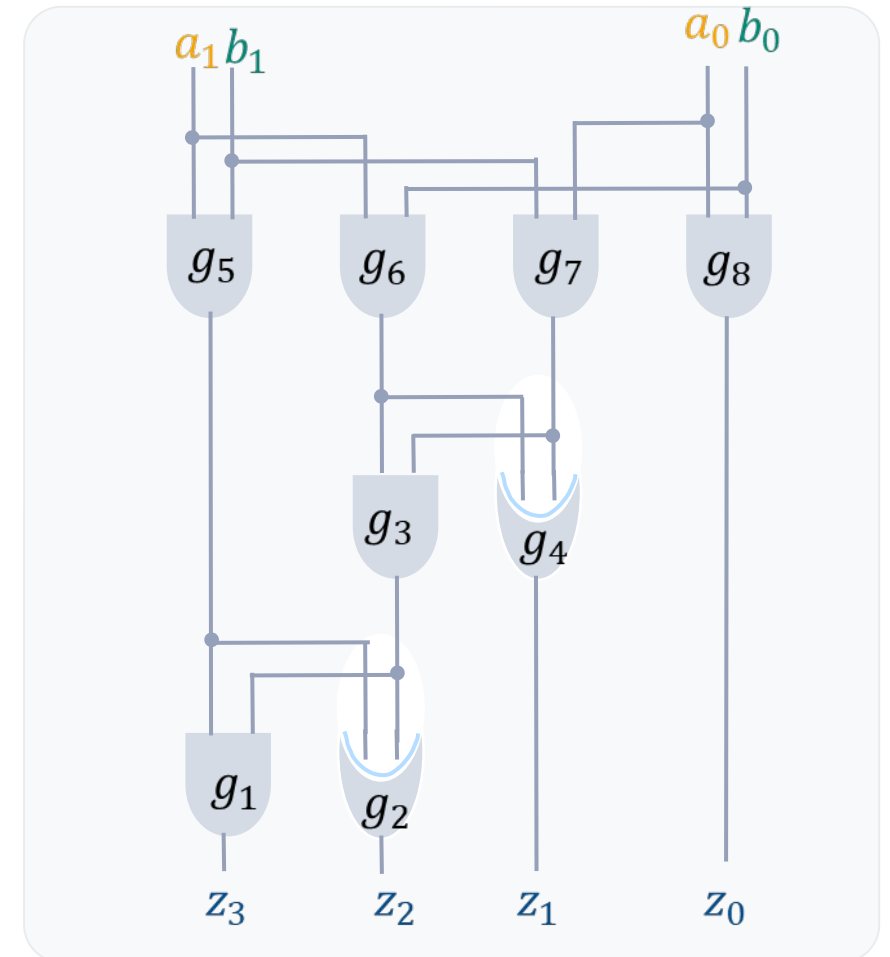
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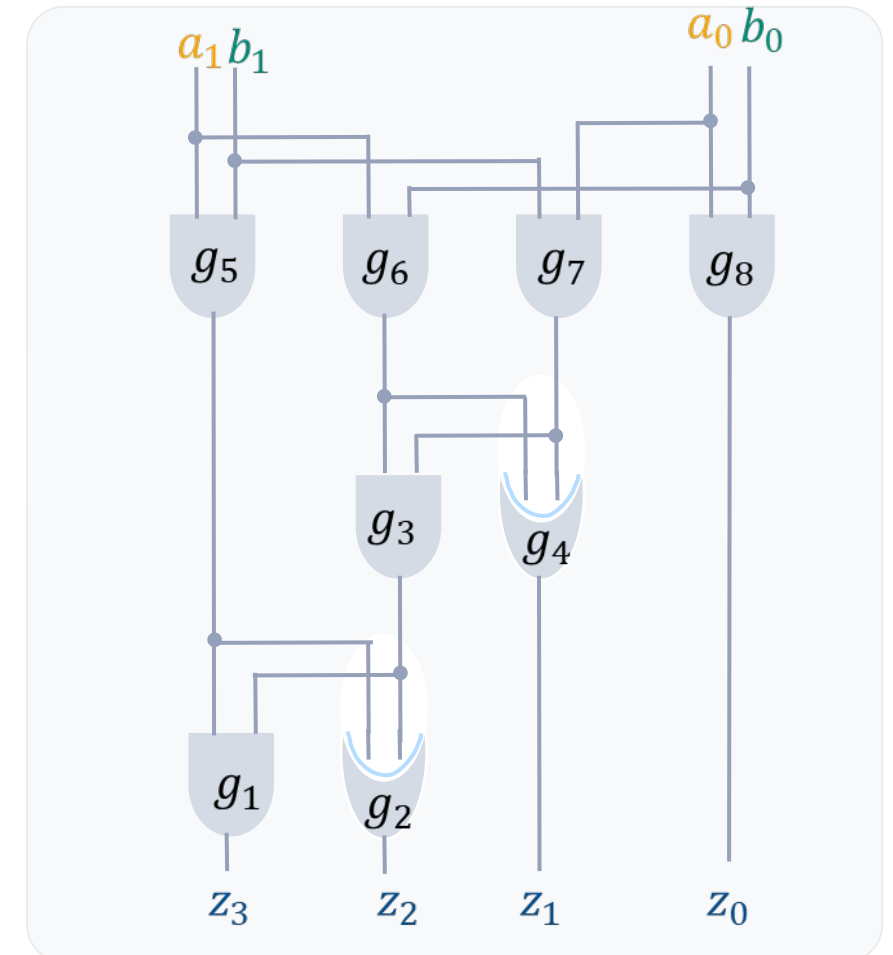
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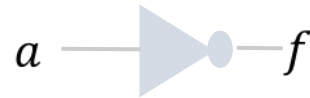
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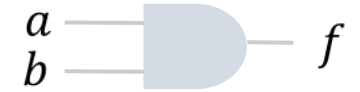
SCA Gate Polynomials

For polynomial representations of gates use word-level interpretation.

Negation: $f = 1 - a$



Conjunction: $f = a \cdot b$



Disjunction: $f = a + b - ab$

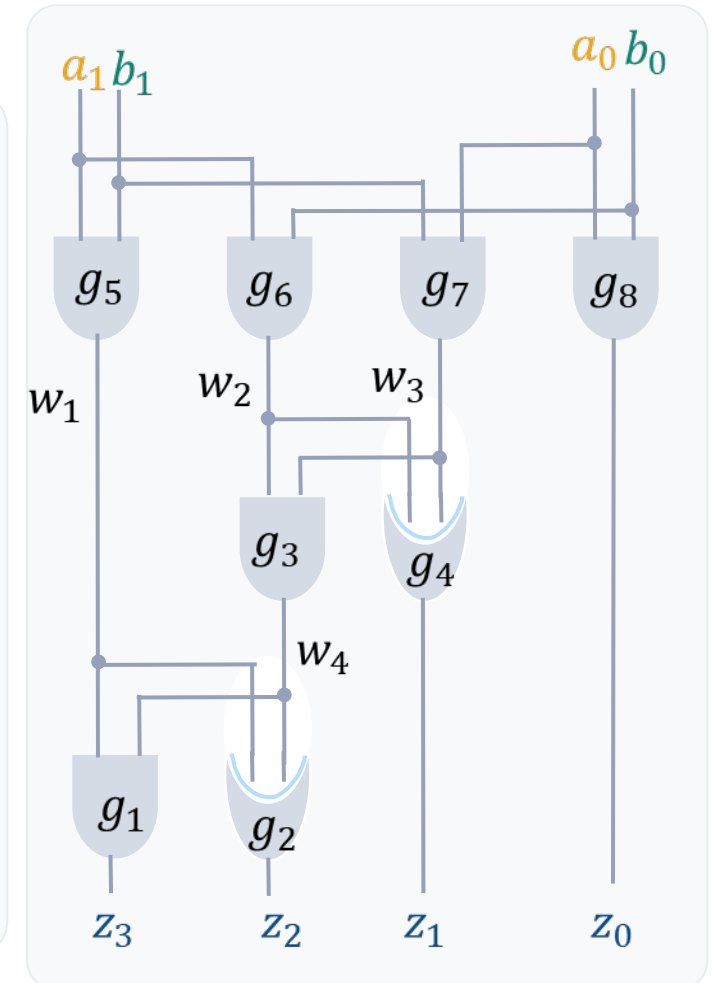


EXOR: $f = a + b - 2ab$



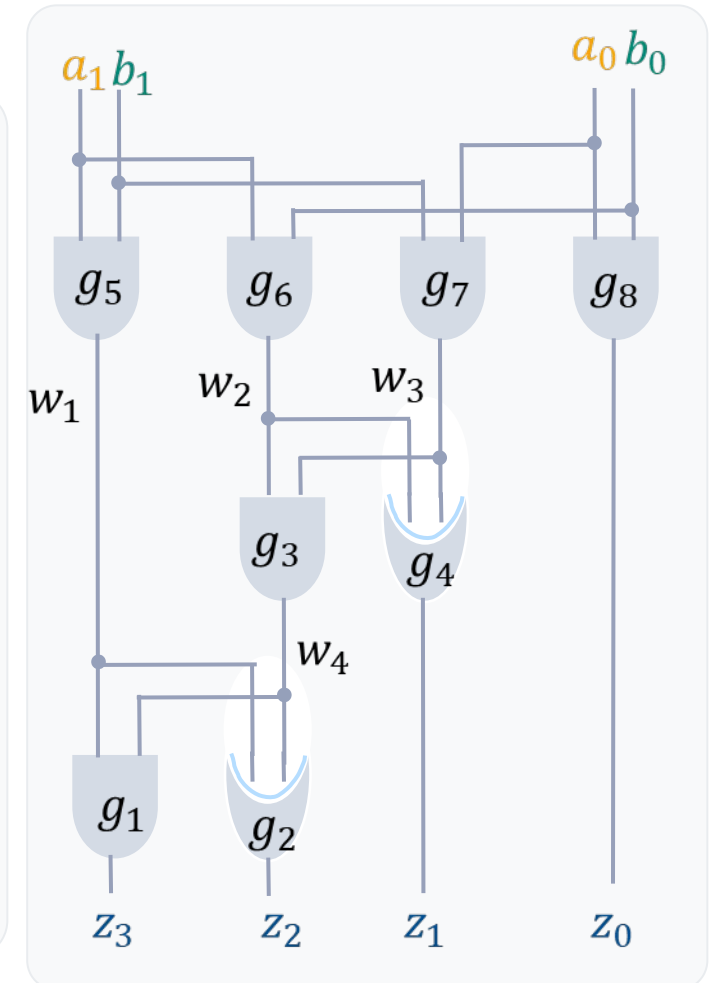
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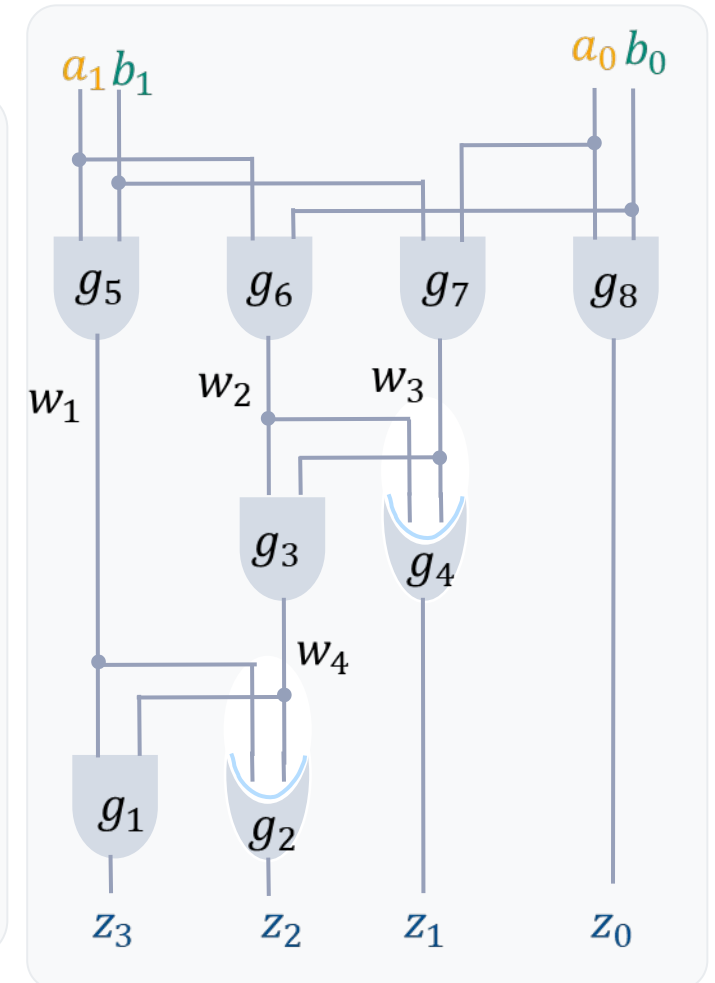
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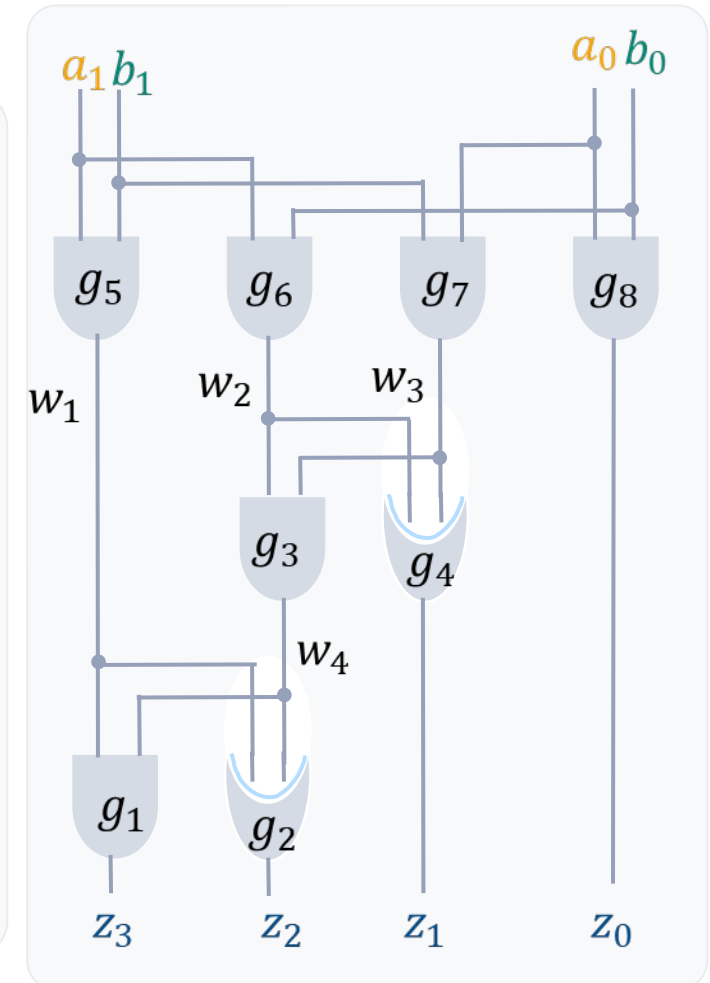
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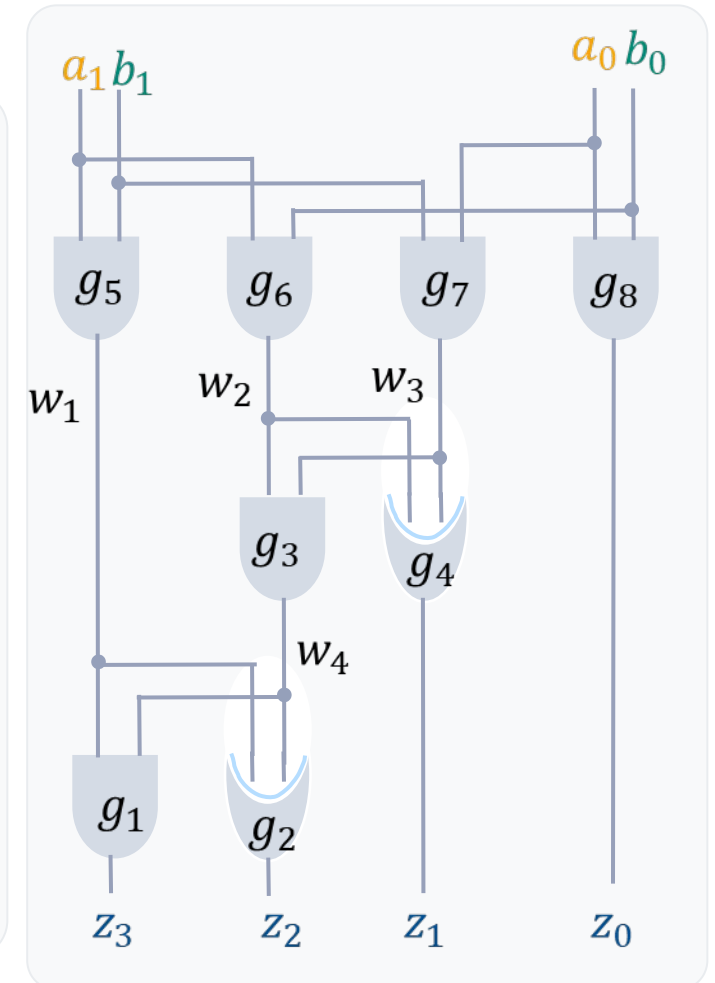


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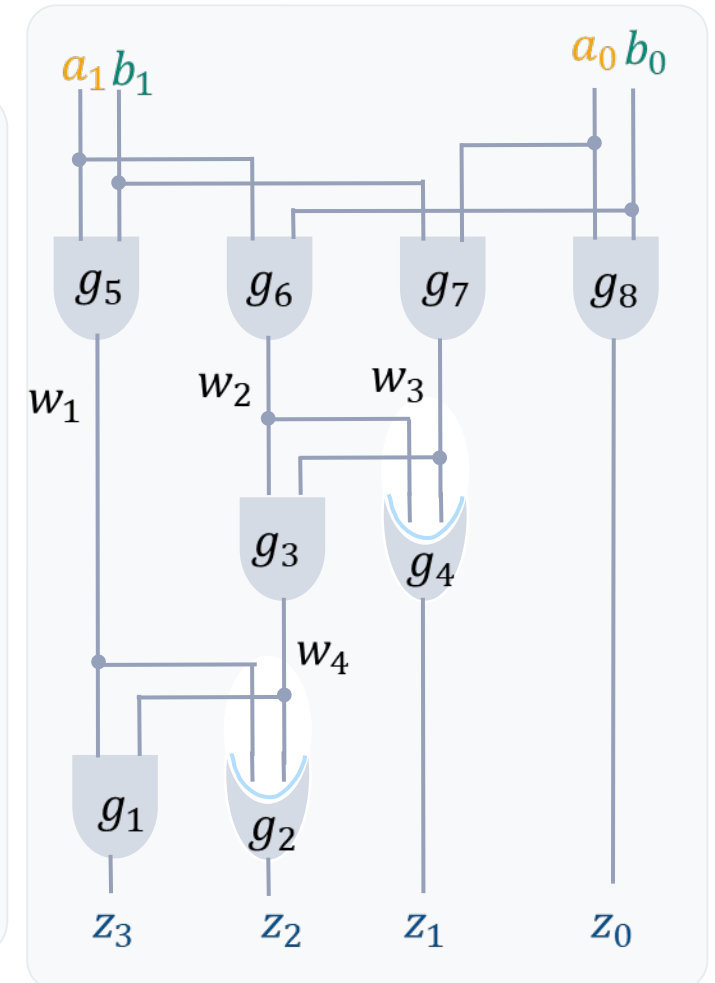


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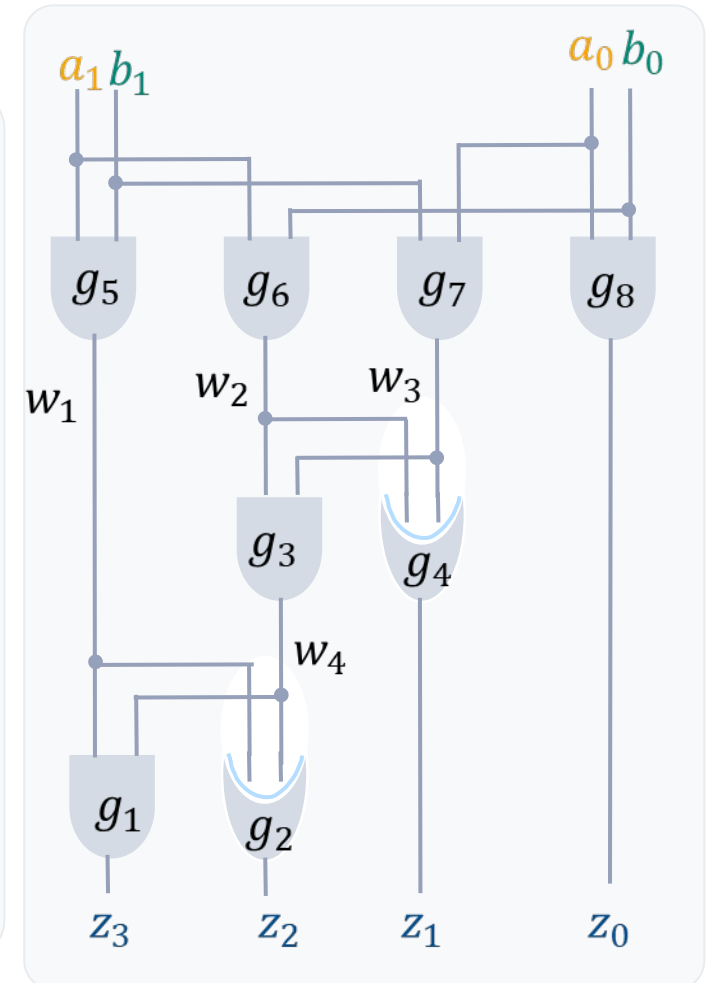


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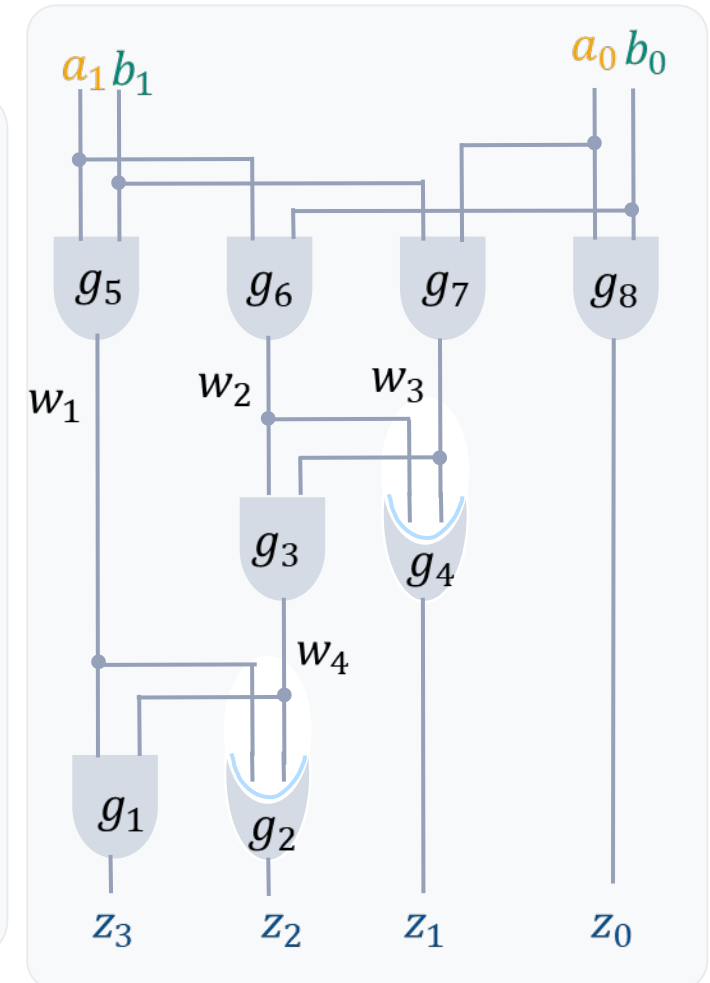


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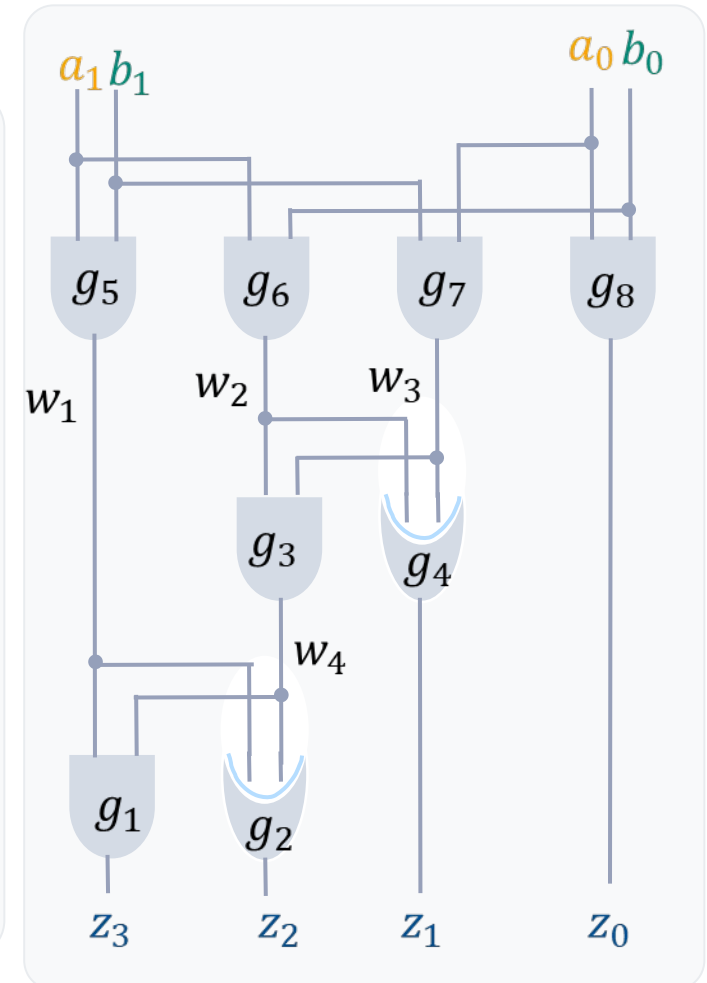
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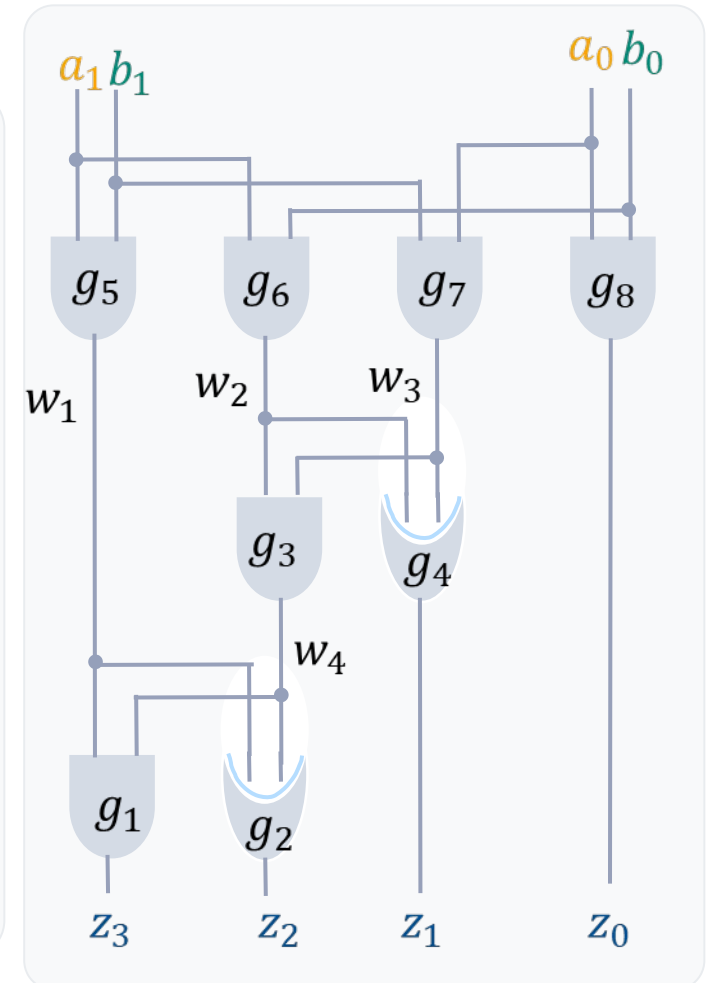
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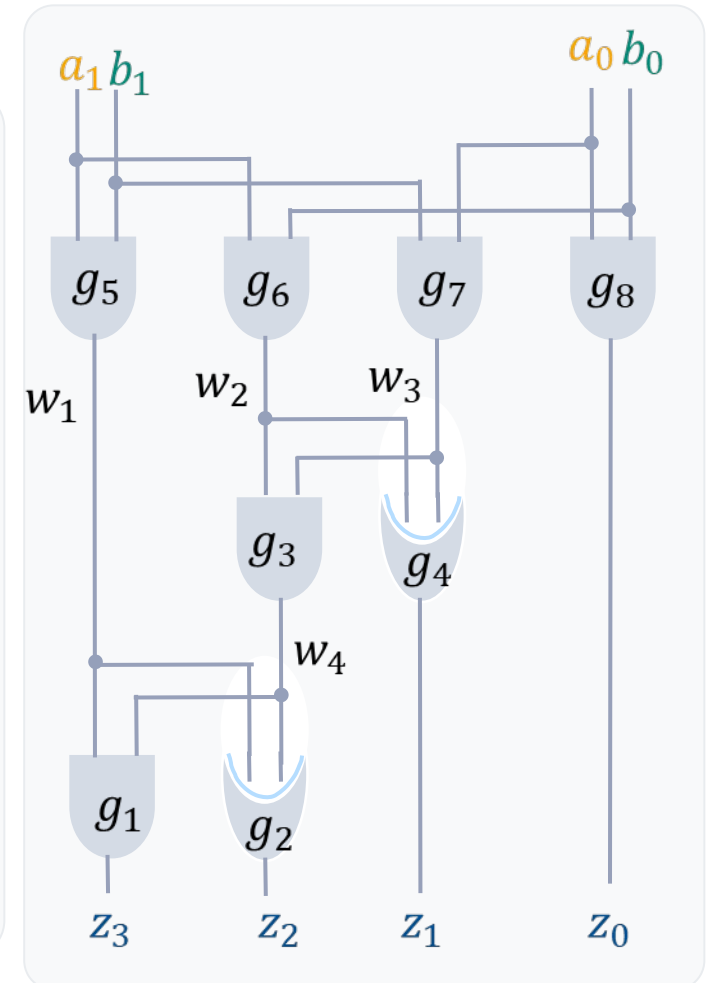
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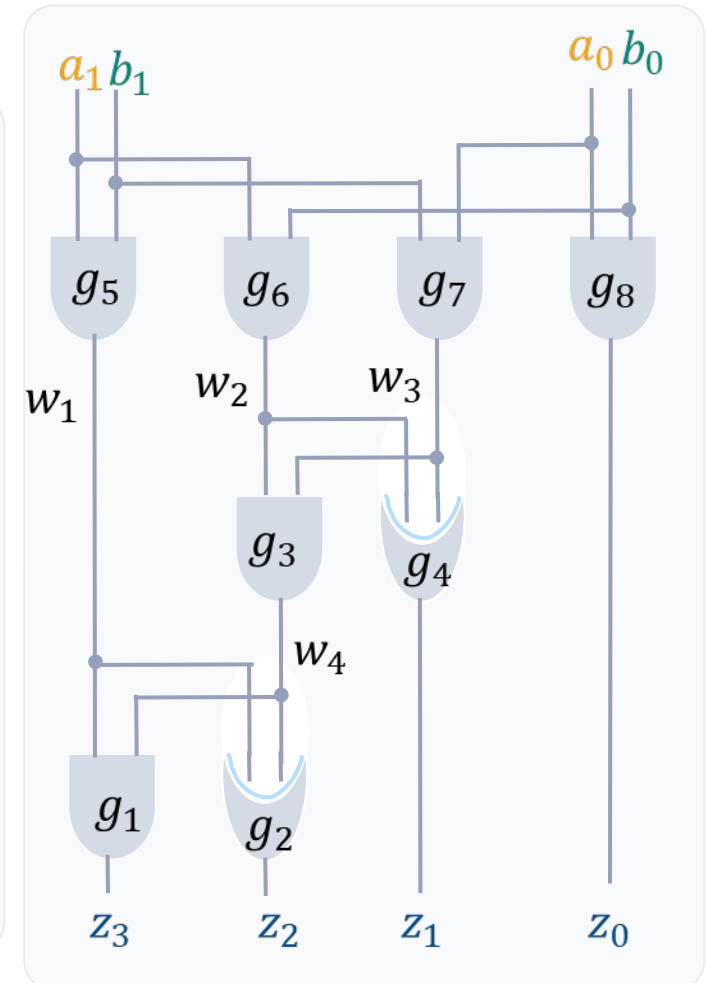
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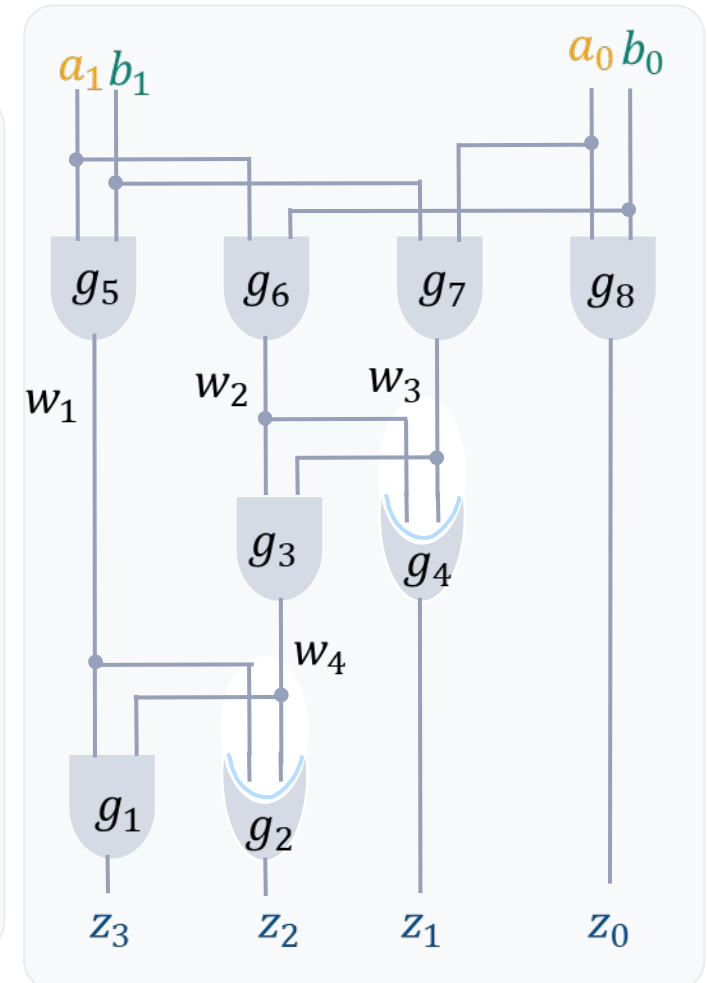
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$$SP_{5,6,7,8} = 4\underline{a_1b_1} + 2\underline{a_1b_0} + 2\underline{a_0b_1} + \underline{a_0b_0} - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0)$$



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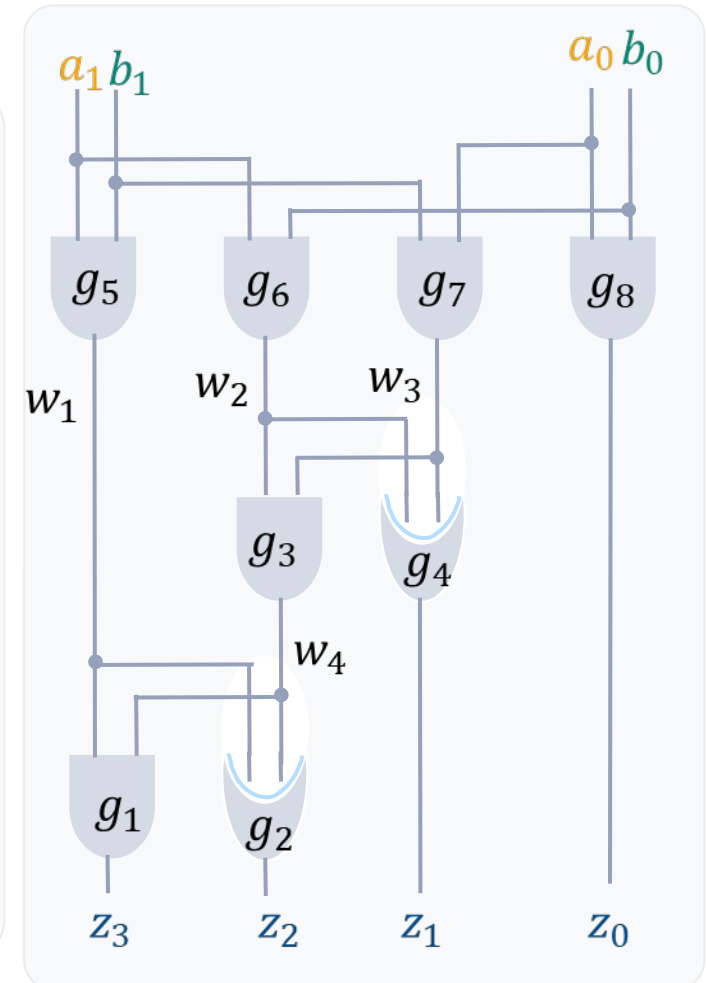
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 \end{aligned}$$

$$SP_{5,6,7,8} = \cancel{4\underline{a_1} \cancel{b_1}} + \cancel{2\underline{a_1} \cancel{b_0}} + \cancel{2\underline{a_0} \cancel{b_1}} + \cancel{a_0 \cancel{b_0}} - (\cancel{4\underline{a_1} \cancel{b_1}} + \cancel{2\underline{a_1} \cancel{b_0}} + \cancel{2\underline{a_0} \cancel{b_1}} + \cancel{a_0 \cancel{b_0}})$$



SCA Term Rewriting

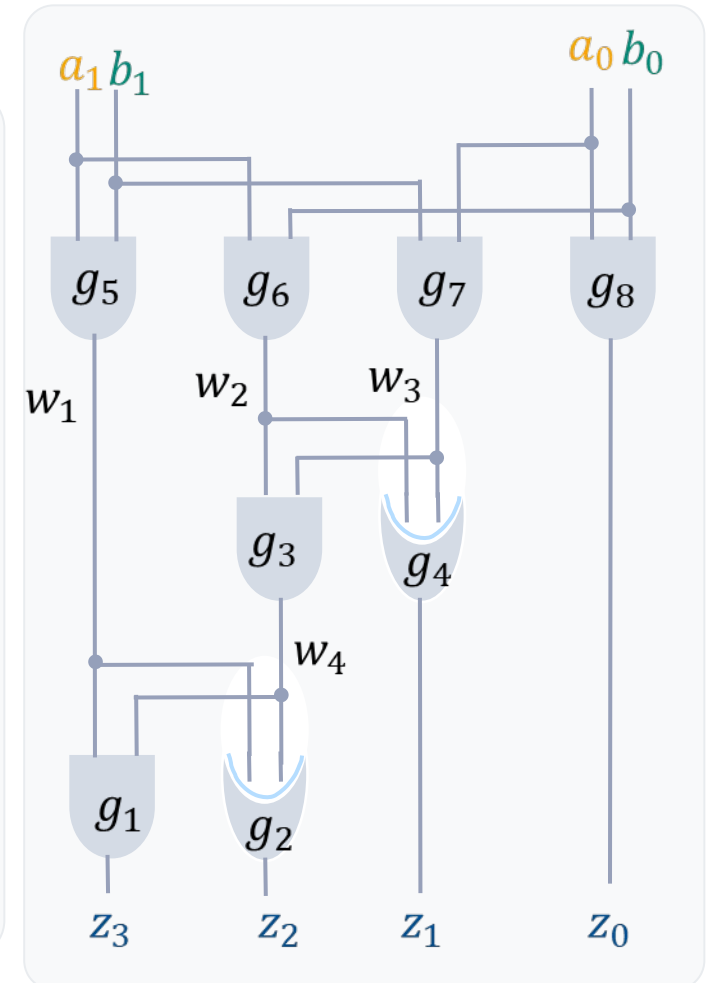
$$\begin{aligned}
 SP &= 8z_3 + 4z_2 + 2z_1 + z_0 - (2a_1 + a_0) \cdot (2b_1 + b_0) \\
 &= 8\underline{z_3} + 4z_2 + 2z_1 + z_0 - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0)
 \end{aligned}$$

$$SP_1 = 8\underline{w_1w_4} + 4\underline{z_2} + 2z_1 + z_0 - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0)$$

$$\begin{aligned}
 SP_2 &= 8\underline{w_1} \cancel{w_4} + 4(\underline{w_1 + w_4 - 2\underline{w_1} \cancel{w_4}}) + 2z_1 + z_0 - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0) \\
 &= 4w_1 + 4\underline{w_4} + 2\underline{z_1} + z_0 - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0)
 \end{aligned}$$

$$\begin{aligned}
 SP_{3,4} &= 4w_1 + 4\underline{w_2} \cancel{w_3} + 2(\underline{w_2 + w_3 - 2\underline{w_2} \cancel{w_3}}) + z_0 - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0) \\
 &= 4\underline{w_1} + 2\underline{w_2} + 2\underline{w_3} + \underline{z_0} - (4a_1b_1 + 2a_1b_0 + 2a_0b_1 + a_0b_0)
 \end{aligned}$$

$$SP_{5,6,7,8} = \cancel{4a_1b_1} + \cancel{2a_1b_0} + \cancel{2a_0b_1} + \cancel{a_0b_0} - (\cancel{4a_1b_1} + \cancel{2a_1b_0} + \cancel{2a_0b_1} + \cancel{a_0b_0}) = 0$$



SCA Optimizations

Problem: Intermediate Term Explosion

Not all functions can be efficiently represented by SCA

Spec Polynomial ✓ Final Polynomial ✓ Intermediate Polynomials ✗

Proposed Optimizations

Dynamic Ordering

Order substitutions dynamically based on their impact on polynomial size to prevent blow-up.

Polarity Optimization

Negate selected variables to choose a more compact arithmetic representation.

Dynamic Ordering Process

The "Look-Ahead" Strategy

- Selects substitutions based on their impact on **polynomial size** to prevent term explosion.
- **Trial Substitutions:** For each available gate in the current cut, a tentative substitution is performed.
- **Decision Logic:**
 - **IF** growth < threshold (1.3): Stop search and apply immediately.
 - **ELSE:** Select the candidate with the smallest relative increase.
- Effectively manages intermediate representations to ensure shorter verification runtimes.

Phase Optimization

Concept: Reducing Monomial Count

- Mitigates excessive polynomial growth by negating selected signals.
- Analogous to Fixed Polarity Reed-Muller expressions at the Boolean level.
- Heuristic evaluation: Flipping signal polarity is checked during substitution.

Example: OR Function

Positive Polarity:

$$a + b + c - ab - ac - bc + 2abc$$

Complemented Form (Compact):

$$1 - \overline{abc}$$

Conflict Optimization

- Assume a circuit representing a function $f: \text{IN} \rightarrow \text{OUT}$
- After substitution until a cut \mathbf{C} , the circuit is divided into:
 - Substituted part $g: \mathbf{C} \rightarrow \text{OUT}$
 - Remaining part $h: \text{IN} \rightarrow \mathbf{C}$
- Intermediate polynomial represents \mathbf{g}
- h is not necessarily injective \Rightarrow polynomial represents \mathbf{g} for irrelevant input combinations

Conflict Optimization

Overview

- In a polynomial each monomial represents a partial variable assignment
- Two signals are in a **conflict** if they cannot evaluate to 1 at the same time
- Monomials containing conflicting variables can be removed \Rightarrow **polynomial size reduction**
- *Example:* Half adder outputs (vanishing monomials introduced by Mahzoon et al. [1])

Challenges

- Calculate conflicting signals
- Remove monomials containing conflicts during substitution

[1] Mahzoon et al., 2018. PolyCleaner: Clean your polynomials before backward rewriting to verify million-gate multipliers. ICCAD '18

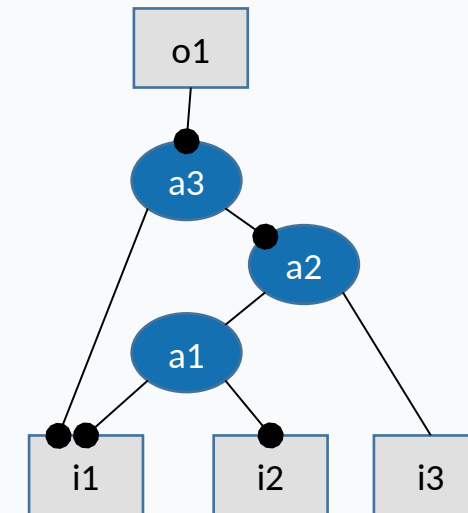
Conflict Calculation Process

Algorithmic Overview

- Pre-processing step via **Implication Propagation**
- Iterates over every signal s and constant value $v \in \{0,1\}$
- Uses **Forward** and **Backward** implication rules
- Identifies **Mutually Exclusive** signal pairs

Example

1



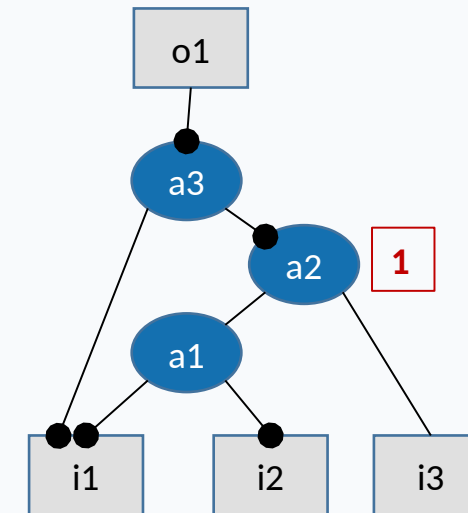
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Example

1



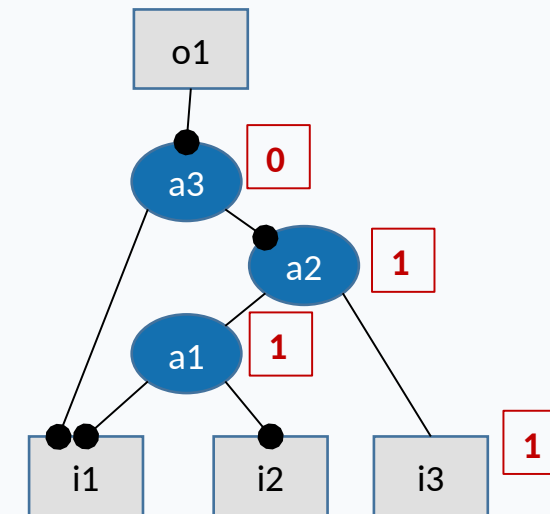
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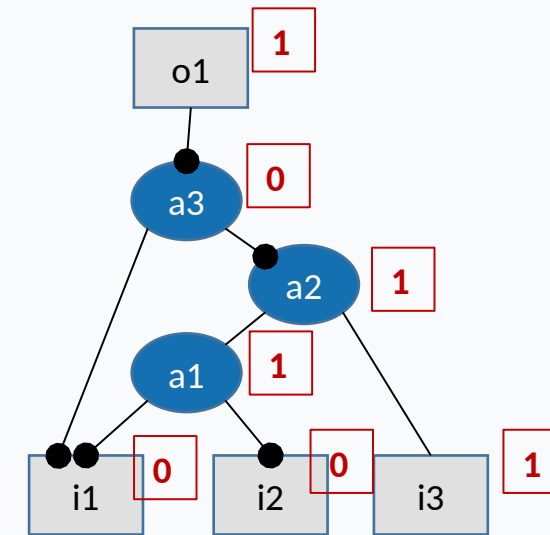
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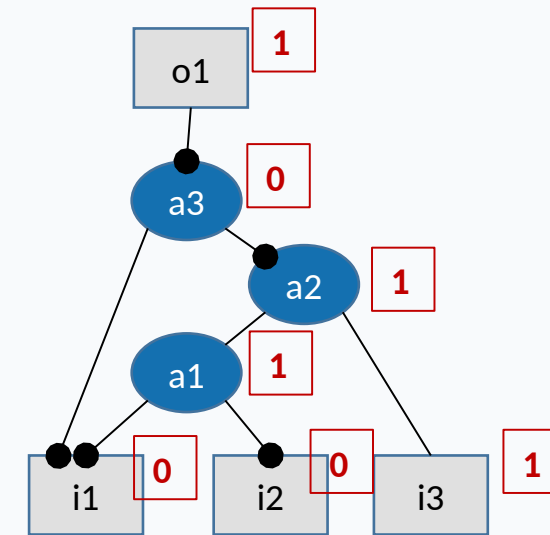
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Example

1



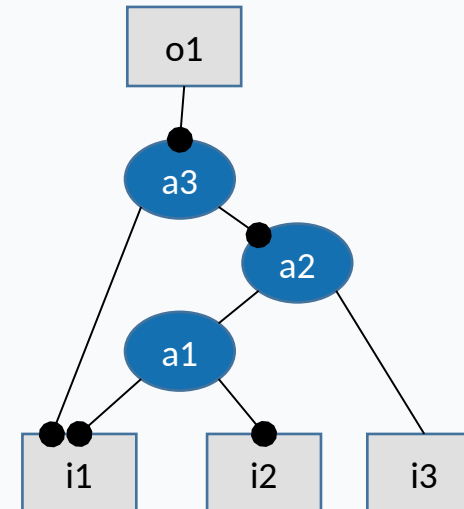
Signal **a2** is in conflict with **a3**, **i1** and **i2**

Conflict Removal

- After each substitution step scan all edited monomials for conflicts
- Conflicts can be stored in map: signal \Rightarrow conflicting signals

Substitution Process

$$\begin{aligned}
 & o1 \\
 =^{o1} & 1 - a3 \\
 =^{a3} & a2 + i1 - \cancel{a2i1} \\
 = & a2 + i1 \\
 =^{a2} & a1i3 + i1 \\
 =^{a1} & i3 - i1i3 - i2i3 + i1i2i3 + i1
 \end{aligned}$$



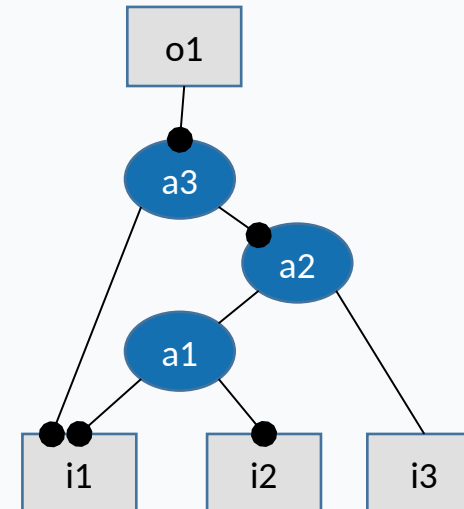
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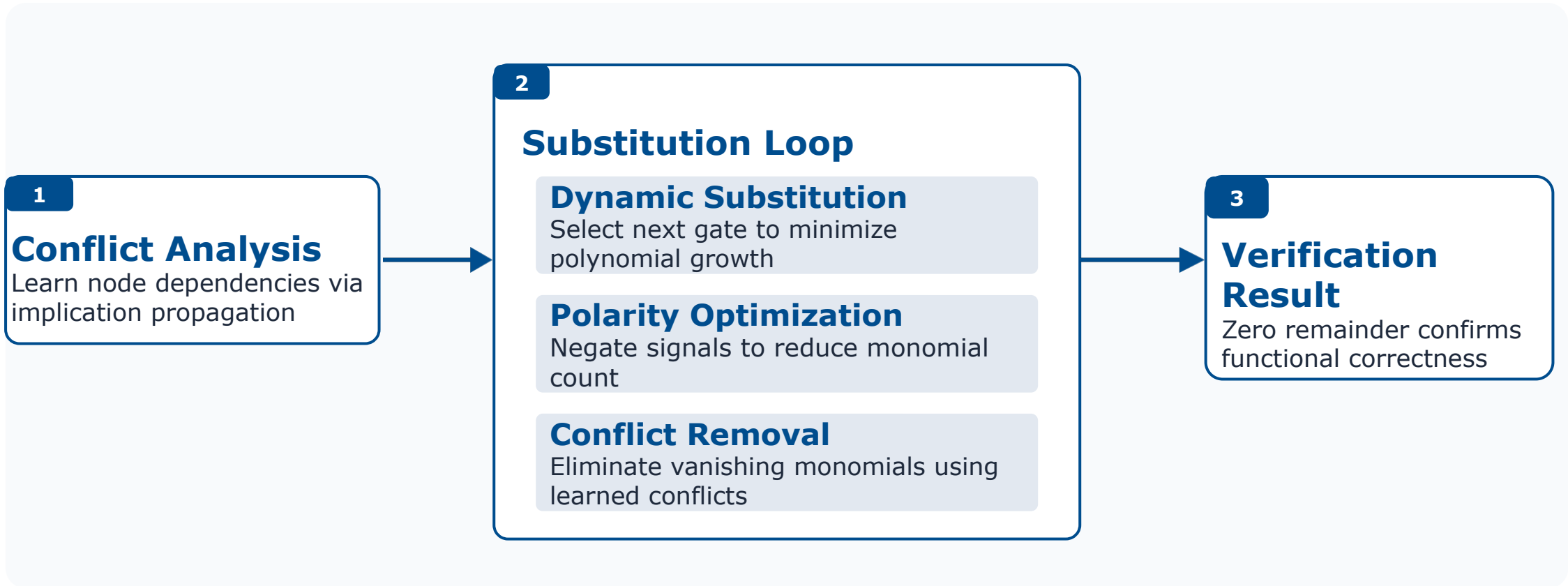
Substitution Process

$$\begin{aligned}
 & o1 \\
 =^{o1} & 1 - a3 \\
 =^{a3} & a2 + i1 - \cancel{a2i1} \\
 = & a2 + i1 \\
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 =^{a1} & i3 - i1i3 - i2i3 + i1i2i3 + i1
 \end{aligned}$$

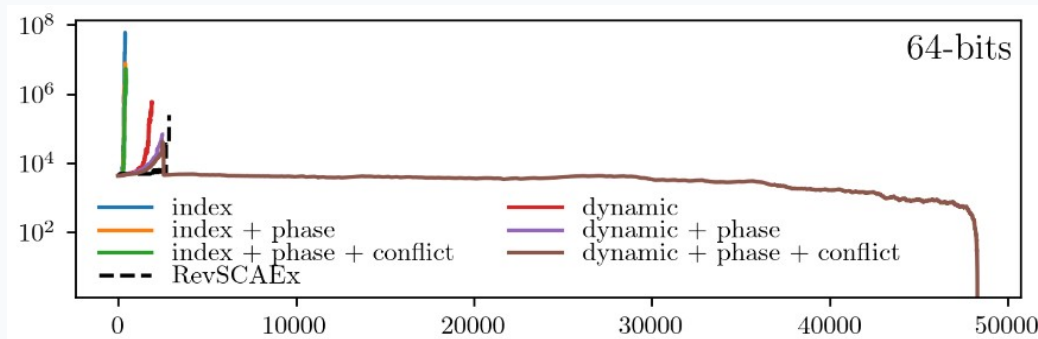
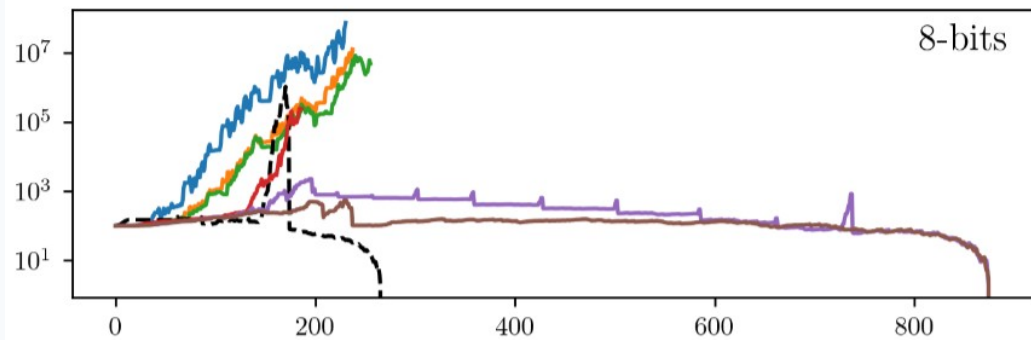
Signal a2 is in conflict with a3, i1 and i2



SCA-Based Verification Engine Design



Configuration Evaluation: WT-KS



Performance Analysis

Key Observations:

- **Dynamic + Phase + Conflict:** Achieves the most stable and compressed intermediate polynomial expressions.
- **Scalability:** Successfully verifies 64-bit designs where baseline and partial optimizations time out.
- **Term Explosion:** Effectively mitigates intermediate term explosion through learned conflicts.

Results on Structural MACs

Design	8-Bit		16-Bit		32-Bit	
	MP	VT	MP	VT	MP	VT
AR_CL	637	0.2928	5874	21.1273	T.O.	T.O.
AR_CS	420	0.0681	522	0.4039	100194	198.9790
AR_KS	172	0.0438	1380	1.5311	5672	97.5673
AR_RC	170	0.0436	543	0.3768	1837	5.9557
DT_CL	815	0.5289	T.O.	T.O.	T.O.	T.O.
DT_CS	261	0.0607	T.O.	T.O.	T.O.	T.O.
DT_KS	476	0.0726	1943	1.1814	26984	148.7580
DT_RC	182	0.0440	678	0.4853	2179	7.3185
WT_CL	4862	4.7639	595692	1334.5800	T.O.	T.O.
WT_CS	246	0.0553	2166	1.1536	T.O.	T.O.
WT_KS	544	0.2005	2206	0.7927	26942	20.8172
WT_RC	170	0.0482	618	0.4587	2136	7.2476
Σ	28/28	28/28 RevSCA	25/28	12/28 RevSCAExt	21/28 RevSCAExt	12/28 RevSCAExt

Optimized MACs

B	NN	Proposed					RevSCAExt		Transform	
		MP	VT	PN	DS	NC	MP	VT	MP	VT
2	38	21	0.002	11	1	14	19	0.001	13	0.001
4	147	48	0.009	42	1	55	49	0.003	34	0.002
6	334	82	0.013	81	1	126	418960	176.248	90	0.009
8	596	138	0.039	133	1	224	5819510	1445.410	336	0.033
10	919	181	0.072	220	1	346	T.O.	T.O.	1261	0.128
12	1312	278	0.179	279	1	664	T.O.	T.O.	T.O.	T.O.
16	2295	458	0.196	436	1	1015	T.O.	T.O.	T.O.	T.O.
32	8870	2698	3.431	1830	1.01	4967	T.O.	T.O.	T.O.	T.O.
64	34521	34274	833.401	7060	1.06	367991	T.O.	T.O.	T.O.	T.O.

Conclusion

Approach

- **Conflicts** are used to reduce intermediate polynomial size
- **Implication** based calculation of conflicts
- **Conflict removal** during substitution
- Embedded in engine using **dynamic substitution** and **phase optimization**

Results

- Conflicts make verification of many MAC architectures feasible
- Performance on structural and optimized designs outperforms state-of-the-art

A Conflict-Aware Learning Approach to SCA Verification for MAC Architectures

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Kamalika Datta, Rolf Drechsler**

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